Matching Theory: K_4-based and $\overline{C_6}$-based Planar Graphs

Nishad Kothari

CSE Department

Indian Institute of Technology – Madras

For several problems in Matching Theory, one may restrict attention to *matching covered graphs* — i.e., connected graphs with the additional property that each edge belongs to some perfect matching. Two types of decompositions — *ear decompositions* and *tight cut decompositions* — play an important role in the study of these graphs.

Lovász (1983) proved that every nonbipartite matching covered graph admits an ear decomposition starting from a bi-subdivision of the complete graph K_4, or from a bi-subdivision of the triangular prism $\overline{C_6}$. This gives rise to two natural problems: Which matching covered graphs are K_4-based (i.e., admit an ear decomposition starting from a bi-subdivision of K_4)? Likewise, which ones are $\overline{C_6}$-based?

In a joint work with U. S. R. Murty (`https://onlinelibrary.wiley.com/doi/full/10.1002/jgt.21882`), we solved the aforementioned problems for planar graphs. At a high-level, our solution comprises two steps: (i) reduce each problem to the case of "bricks" (special nonbipartite matching covere graphs) by applying the tight cut decomposition, and (ii) solve each problem for the case of planar bricks.

I will discuss each of these problems, and our solutions for the planar case.